

Joint Modeling Tumor Burden and Time to Event Data in Oncology Trials

Ye Shen, Aparna Anderson, Ritwik Sinha, and Yang Li

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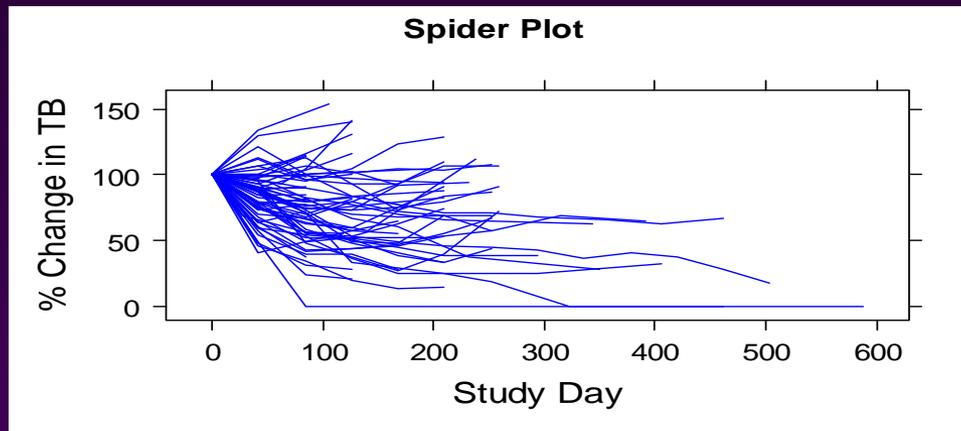
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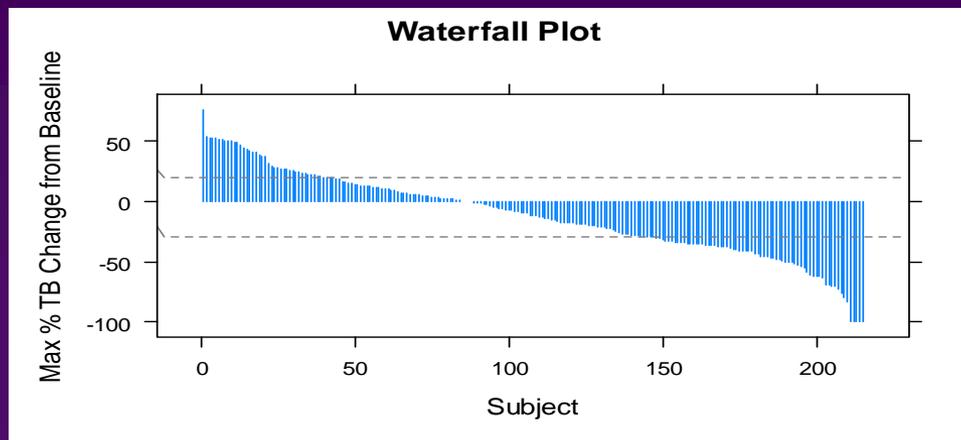
Introduction

- In oncology, overall survival is the ideal measure of treatment benefit
- However, the mechanistic and biologic effects of a therapeutic agent are generally described in terms of **tumor burden (TB)**, measured repeatedly at protocol-specified time intervals
- Tumor burden is usually categorized (e.g., per RECIST) for the purpose of analyzing objective response rate or **progression-free survival (PFS)**
- The loss of information due to categorization may result in a misrepresentation of the true association between treatment and change in TB
- **Question: How can we fully exploit the complete longitudinal tumor burden data to characterize biological treatment effects?**

Commonly applied methods have important limitations



- Patterns obscured if too many subjects
- Qualitative; no formal inference
- Bias due to lack of follow-up in those who progress or die (non-random missingness)

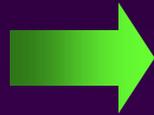


- A different measure of change (e.g., max % change vs. nadir) might reveal a different pattern
- Qualitative; no formal inference
- Ignores timing, durability, and survival

The nature of missing TB data affects estimation and interpretability

Missing completely at random (MCAR):

Missing value does not depend on observed or unobserved measurements of interest



Example:

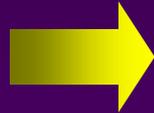
Subjects are more likely to miss tumor assessment visits during the holiday season



Loss of power

Missing at random (MAR):

Missing value does not depend on unobserved value itself, but possibly on other factors



Example:

Subjects have missing TB data following death or evidence of disease progression

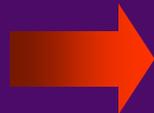


Bias

Loss of power

Missing not at random (MNAR):

Missing value depends on unobserved value itself



Example:

Subjects with high TB are more likely to miss tumor assessment visits due to impaired mobility



Loss of efficiency

A more robust method is required

- We want a model that
 - uses the **longitudinal TB data**
 - accounts for **MAR and possible MNAR** due respectively to PFS and a possible latent missing mechanism
 - allows **formal inference** about the association between treatment and TB
 - We want to evaluate how such a model performs
 - with respect to bias and variance
 - under linear and nonlinear tumor burden distributions
- as compared to
- a gold standard model fitted to fully observed data (i.e., no missingness)
 - a simpler model that requires missing data to be (at worst) MAR

Construct a model that jointly captures TB and PFS

- **Linear mixed effects (LME)** model for longitudinal TB data

$$TB(t) = \text{covariate}(t) * \beta_1 + RE_1(t) + \text{error}$$

- **Event time (ET)** model for progression-free survival

$$\text{hazard}(t) = \exp \{ \text{covariate}(t) * \beta_2 + RE_2 \}$$

PFS is defined as the time from study entry to disease progression or death, whichever occurs first

- **Joint model (JM):** Introduce correlation between random effects (RE_1, RE_2):

$$RE_1(t) = U_1 + U_2 * t \quad \text{and} \quad RE_2 = \pi_1 * U_1 + \pi_2 * U_2$$

where U_1 = random intercept, U_2 = random slope, and π_1, π_2 = fixed coefficients

NOTE: Non-zero π_1 and π_2 produces MNAR data

LME alone offers a simple alternative to the more complex JM, but LME gives biased results if MNAR conditions exist (Schafer and Graham, 2000).

Estimate joint model parameters

- A joint likelihood function can be constructed for the LME and ET components of the JM
 - L_Y = likelihood of longitudinal TB process
 - $L_{N|Y}$ = conditional likelihood of event process for progression-free survival, given TB data
 - $L = L_Y \times L_{N|Y}$ = joint likelihood function (A)
(Henderson et al., 2000)
- Apply algorithm and SAS program developed by Guo and Carlin (2004) to maximize joint likelihood

Simulation 1: JM operating characteristics under linear TB

- Apply (B) to simulate data:

$$TB(t) \sim b_{L0} + b_{L1} * t + b_{L2} * t * Trt + U_1 + U_2 * t + \text{error}$$

$$\text{Log(PFS hazard)} \sim b_{S0} + b_{S1} * Trt + p_1 * U_1 + p_2 * U_2 \quad (\text{B})$$

where $U_1 \sim N(0, v_{11})$, $U_2 \sim N(0, v_{22})$, $\text{error} \sim N(0, 1)$, and

b_{L0} , b_{L1} , b_{L2} , b_{S0} , b_{S1} are fixed at 0.4, 0.1, -0.2, -4, -1, respectively

- TB values q6wks in $t=[0,48]$, $n=300$, replications=200
- Missing mechanism for data generated from (B):
 - If event occurs on or before $t=48$, set TB data as missing after event time
 - If event occurs after $t=48$, no missing TB and censor PFS at $t=48$

Simulation 1:

Linear TB, missing data are MAR

Parameter estimates (SD) for b_{L0} , b_{L1} , and b_{L2} from TB model

	FLME	PLME	PJM	FLME	PLME	PJM
True parameters	$p_1 = 0, p_2 = 0$ (MAR)					
	$v_{11} = 0.1, v_{22} = 0.05$			$v_{11} = 1, v_{22} = 0.05$		
$b_{L0} = 0.4$	0.40(0.029)	0.40(0.028)	0.39(0.033)	0.40(0.070)	0.40(0.071)	0.40(0.072)
$b_{L1} = 0.1$	0.10(0.005)	0.10(0.006)	0.10(0.006)	0.10(0.005)	0.10(0.006)	0.10(0.007)
$b_{L2} = -0.2$	-0.20(0.005)	-0.20(0.006)	-0.20(0.007)	-0.20(0.005)	-0.20(0.006)	-0.20(0.007)
	$v_{11} = 0.1, v_{22} = 0.5$			$v_{11} = 1, v_{22} = 0.5$		
$b_{L0} = 0.4$	0.40(0.039)	0.40(0.038)	0.40(0.038)	0.40(0.061)	0.40(0.066)	0.41(0.068)
$b_{L1} = 0.1$	0.09(0.055)	0.09(0.057)	0.09(0.057)	0.09(0.061)	0.08(0.059)	0.09(0.060)
$b_{L2} = -0.2$	-0.19(0.036)	-0.19(0.037)	-0.19(0.038)	-0.20(0.054)	-0.20(0.051)	-0.20(0.053)

FLME = LME fitted on full data (ideal) PLME = LME fitted on partial data due to MAR PJM = Joint Model fitted on partial data

Conclusion: Under MAR, JM produces unbiased estimates and shows minimal loss of efficiency compared to correctly specified LME approach.

Simulation 1:

Linear TB, missing data are MNAR

Parameter estimates (SD) for b_{L0} , b_{L1} , and b_{L2} from TB model

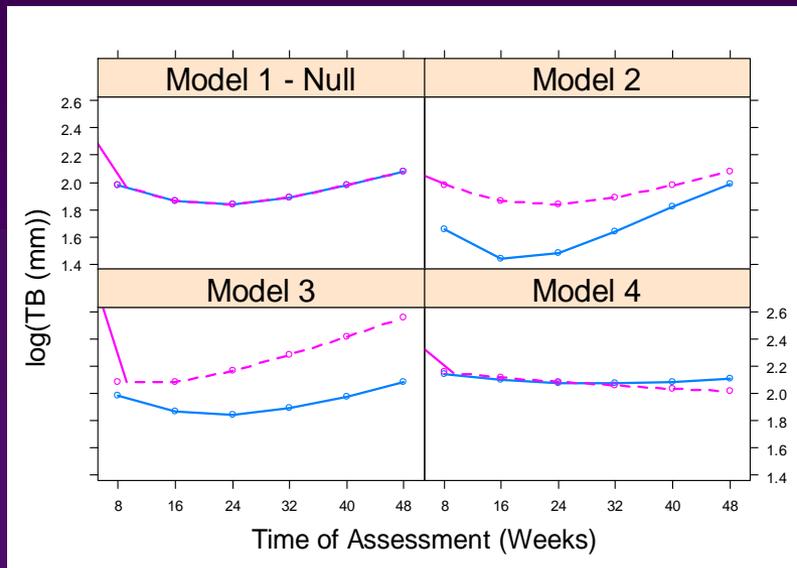
	FLME	PLME	PJM	FLME	PLME	PJM
True parameters	$p_1 = -1.5, p_2 = 2$ (MNAR)					
	$v_{11} = 0.1, v_{22} = 0.05$			$v_{11} = 1, v_{22} = 0.05$		
$b_{L0} = 0.4$	0.40(0.033)	0.39(0.037)	0.39(0.039)	0.39(0.070)	0.33(0.071)	0.40(0.072)
$b_{L1} = 0.1$	0.10(0.006)	0.10(0.006)	0.10(0.007)	0.10(0.007)	0.10(0.008)	0.10(0.008)
$b_{L2} = -0.2$	-0.20(0.004)	-0.20(0.005)	-0.20(0.006)	-0.20(0.004)	-0.20(0.005)	-0.20(0.006)
	$v_{11} = 0.1, v_{22} = 0.5$			$v_{11} = 1, v_{22} = 0.5$		
	$b_{L0} = 0.4$	0.40(0.035)	0.39(0.038)	0.40(0.039)	0.40(0.062)	0.29(0.066)
$b_{L1} = 0.1$	0.10(0.054)	0.05(0.056)	0.09(0.061)	0.10(0.052)	0.06(0.059)	0.10(0.060)
$b_{L2} = -0.2$	-0.19(0.038)	-0.11(0.040)	-0.19(0.044)	-0.20(0.042)	-0.09(0.046)	-0.20(0.052)

FLME = LME fitted on full data (ideal) PLME = LME fitted on partial data due to MNAR PJM = Joint Model fitted on partial data

Conclusion: Under MNAR and large inter-subject variability, JM reduces estimation bias compared to misspecified LME model.

Simulation 2: JM operating characteristics under non-linear TB

- Consider **non-linear TB** distribution (Wang et al., 2009)
- Non-linear TB curves reflect an initial dip due to tumor shrinkage, followed by an increase (progression) as the treatment loses its effectiveness
- MNAR mechanism; TB values q8wks in $t=[0,48]$; $n=300$; 200 replications



TB data generated under 4 scenarios:

- **Model 1:** No treatment effect (null model)
- **Model 2:** TB shrinks more quickly in one arm, but progression occurs at the same rate in both arms
- **Model 3:** Both arms have the same shrinkage rate, but different rates of progression
- **Model 4:** One arm has slow steady decrease in TB (phenomenon seen with immunotherapies), while other has progression

Simulation 2:

Non-linear TB, missing data are MNAR

Parameter estimates (Bias) for b_{L0} , b_{L1} , and b_{L2} from TB model

	FLME †	Bias(PLME)	Bias(PJM)	FLME †	Bias(PLME)	Bias(PJM)
	Model 1 (Null condition – no difference)			Model 2 (Different shrinkage rates)		
\hat{b}_{L0}	1.37	1.17	0.80	1.24	0.75	0.51
\hat{b}_{L1}	-0.002	0.137	0.089	-0.002	0.060	0.059
\hat{b}_{L2}	0	0.017	0.004	-0.010	0.048	0.019
	Model 3 (Different progression rates)			Model 4 (Stability vs. progression)		
\hat{b}_{L0}	1.45	1.24	0.84	1.00	0.90	0.65
\hat{b}_{L1}	0.005	0.151	0.095	-0.006	0.160	0.084
\hat{b}_{L2}	-0.001	0.021	0.004	0.014	0.112	0.022

† FLME is a misspecified model in Simulation 2 and therefore not the ideal reference. Convergence issues with non-linear methods prevented calculation of gold standard estimates.

Conclusion: With estimates from FLME as points of reference, JM introduces less bias than LME by accounting for latent MNAR process.

Two-stage piecewise linear mixed effects model

- Longitudinal data generated following the non-linear TB component suggested by Wang et. al.(2009). Event defined as 20% increase in TB from baseline or nadir.
- To account for the non-linear trajectory in TB, the two stages were suggested by previous simulation results from the figure.

- **Stage 1: $t = [0, 24]$**

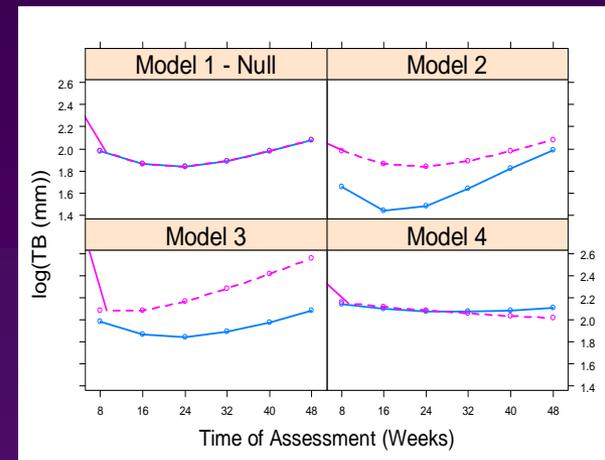
$$TB(t) \sim b_{L01} + b_{L11} * t + b_{L21} * t * Trt + U_{11} + U_{21} * t + \text{error}$$

$$\text{Log(PFS hazard)} \sim b_{S01} + b_{S11} * Trt + p_{11} * U_{11} + p_{21} * U_{21}$$

- **Stage 2: $t = (24, 48]$**

$$TB(t) \sim b_{L02} + b_{L12} * t + b_{L22} * t * Trt + U_{12} + U_{22} * t + \text{error}$$

$$\text{Log(PFS hazard)} \sim b_{S02} + b_{S12} * Trt + p_{12} * U_{12} + p_{22} * U_{22}$$



- In practice, one may need to check the longitudinal process pattern before making a decision on whether a piecewise linear approximation is needed..

Simulation results from the Two-stage model under MNAR (Stage1)

Parameter estimates (Bias) for b_{L0} , b_{L1} , and b_{L2} from TB model						
	FLME †	Bias(PLME)	Bias(PJM)	FLME †	Bias(PLME)	Bias(PJM)
	Model 1 (Null condition – no difference)			Model 2 (Different shrinkage rates)		
\hat{b}_{L01}	2.136	0.017	0.011	2.111	0.023	0.017
\hat{b}_{L11}	-0.016	4.7E-3	3.2E-3	-0.016	5.9E-3	4.0E-3
\hat{b}_{L21}	-6.7E-4	8.2E-4	6.4E-4	-6.2E-3	1.3E-3	1.2E-3
	Model 3 (Different progression rates)			Model 4 (Stability vs. progression)		
\hat{b}_{L01}	2.132	0.019	0.013	2.169	5.5E-3	4.3E-3
\hat{b}_{L11}	-0.017	4.8E-3	3.2E-3	-7.3E-3	3.3E-4	1.4E-4
\hat{b}_{L21}	0.012	1.6E-3	1.2E-3	-0.021	2.2E-3	2.0E-3

† FLME is estimated through two-stage piecewise linear approximation.

Simulation results from the Two-stage model under MNAR (Stage2)

Parameter estimates (Bias) for b_{L0} , b_{L1} , and b_{L2} from TB model

	FLME †	Bias(PLME)	Bias(PJM)	FLME †	Bias(PLME)	Bias(PJM)
	Model 1 (Null condition – no difference)			Model 2 (Different shrinkage rates)		
\hat{b}_{L02}	1.502	0.034	0.011	1.433	0.037	0.012
\hat{b}_{L12}	6.4E-3	4.1E-3	1.4E-3	6.4E-3	4.0E-3	1.3E-3
\hat{b}_{L22}	-2.6E-4	9.1E-4	3.5E-4	2.0E-3	1.3E-3	5.4E-4
	Model 3 (Different progression rates)			Model 4 (Stability vs. progression)		
\hat{b}_{L02}	1.570	0.037	0.011	1.701	0.013	4.8E-3
\hat{b}_{L12}	5.7E-3	4.2E-3	1.4E-3	-5.5E-3	4.4E-4	8.8E-5
\hat{b}_{L22}	2.7E-3	1.0E-3	4.3E-4	8.0E-3	2.6E-3	1.2E-3

† FLME is estimated through two-stage piecewise linear approximation.

Conclusion:

More reduction in Bias observed in Stage2 due to more drop-outs.

Comparisons between one-stage and two-stage joint models

- **Goodness-of-fit Comparisons between one-stage and two-stage joint models**

	Model 1		Model 2		Model 3		Model 4	
	one-stage	two-stage	one-stage	two-stage	one-stage	two-stage	one-stage	two-stage
	AMSE*	0.048	0.016	0.061	0.024	0.046	0.017	0.024

*AMSE is the averaged mean squared error between the observed and predicted main outcomes

Conclusions

- Under MAR and linear TB, JM produces unbiased estimates and shows minimal loss of efficiency compared to the correctly specified LME
- Under MNAR, linear TB, and large inter-subject variability, JM reduces estimation bias compared with the misspecified LME
- Under MNAR and non-linear TB distributions, JM appears to have more favorable operating characteristics compared to LME
- Further exploration is required to incorporate non-linear TB modeling and corresponding estimation algorithms

References

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Schafer, J. L. and Graham, J. W. (2002). "Missing data: Our view of the state of the art," *Psychological Methods*, 7, 147-177

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Thank you!

Appendix: Wang et. al. (2009)'s non-linear TB model

- Wang et. al. (2009) validated this model in four Phase III studies and provided estimates of the parameters. This relatively flexible model offers a wide range of plausible trajectories for TB data.

- The following notation is used to explain the model.

$TSO_i(t)$ = Observed tumor burden of subject i at time t

$TS_i(t)$ = Tumor burden of subject i at time t

$Base_i$ = Baseline tumor burden of subject i

SR_i = Tumor Shrinkage Rate of subject i

PR_i = Progression Rate of subject i

ϵ = Random measurement error

- Then the no-linear longitudinal TB model is defined by the equations:

$$TS_i(t) = Base_i \times \exp(-SR_i \times t) + PR_i \times t$$

$$TSO_i(t) = TS_i(t) \times \exp(\epsilon)$$